Killara HIGH SCHOOL

Student Number

2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Extension 1 Mathematics

25th July 2014

General Instructions

- Reading time – 5 minutes
- Working tine 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages 2 - 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 6 - 11*

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment

Section I

10marks
Attempt Question 1 – 10
Allow about 15 minutes for this section

Use the multiple – choice answer sheet for Questions 1 – 10

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1 What is the natural domain of $f(x) = x + log_e x$?

(A) $x > 0$

(B) All real x

(C) $x > 1$

(D) $x > -1$

2 For the function $f(x) = secx$, where $0 < x < \frac{\pi}{2}$ and $y \geq 1$, What is $f^{-1}(x)$?

(A) $f^{-1}(x) = \frac{1}{\cos^{-1} x}$

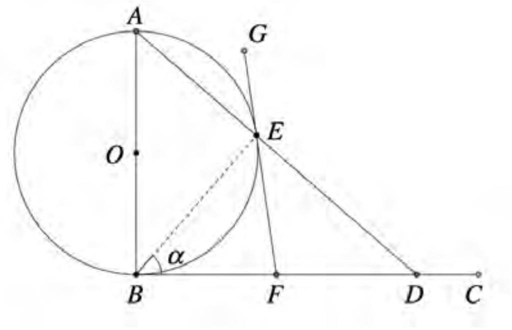
(B) $f^{-1}(x) = \cos^{-1} x$

(C) $f^{-1}(x) = \cos^{-1} \frac{1}{x}$

(D) $f^{-1}(x) = \tan^{-1} x$

3 In the diagram FB and FG are tangents to the circle centre O. $\angle EBF = \alpha$.

What is the size of $\angle FED$?



(A) α

(B) $\dfrac{\alpha}{2}$

(C) $\pi - 2\alpha$

(D) $\dfrac{\pi}{2} - \alpha$

4 The definite integral $\displaystyle\int_{e^3}^{e^4} \frac{1}{x\log_e x}\,dx$ can be written in the form $\displaystyle\int_a^b \frac{1}{u}\,du$ where

(A) $u = \log_e x,\ a = \log_e 3,\ b = \log_e 4$

(B) $u = \log_e x,\ a = 3,\ b = 4$

(C) $u = \log_e x,\ a = e^3,\ b = e^4$

(D) $u = \dfrac{1}{x},\ a = e^{-3},\ b = e^{-4}$

5 Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\frac{x}{2}}$

(A) $\dfrac{3}{2}$

(B) $\dfrac{2}{3}$

(C) 6

(D) $\dfrac{1}{6}$

6 Find the value (in simplest surd form) of $\cos 15°$:

(A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(B) $\dfrac{\sqrt{2}-\sqrt{3}}{2}$

(C) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(D) $\dfrac{\sqrt{2}+\sqrt{6}}{2}$

7 $f(x) = \sqrt{4-x^2}, -2 \le x \le 0.$ The inverse function is give by:

(A) $x^2 + y^2 = 4$

(B) $y = \sqrt{4-x^2}, -2 \le x \le 2$

(C) $y = \sqrt{4-x^2}, 0 \le x \le 2$

(D) $y = -\sqrt{4-x^2}, 0 \le x \le 2$

8 The solution to $\sqrt{2}\sin 2\theta + 1 = 0$ for $0 \le \theta \le 2\pi$ is:

(A) $\dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

(B) $\dfrac{5\pi}{8}, \dfrac{7\pi}{8}$

(C) $\dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

(D) $\dfrac{5\pi}{4}, \dfrac{7\pi}{4}, \dfrac{13\pi}{4}, \dfrac{15\pi}{4}$

9 If $t = \tan\frac{\theta}{2}$, then $\sin\theta + \cos\theta = \cdots$

(A) $\frac{1+2t-t^2}{1+t^2}$

(B) $\frac{t^2-2t+1}{1+t^2}$

(C) $\frac{(1-t)^2}{1+t^2}$

(D) $\frac{(1+t)^2}{1+t^2}$

10 One solution of the equation $2\cos 2x = x - 1$ is close to $x = 0.9$. Use one application of Newton's method to find another approximation to this solution. Give your answer correct to three decimal places

(A) 0.799

(B) 0.828

(C) 0.909

(D) 0.938

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Factorise (fully) $\left(a^2+ab\right)^2-\left(ab+b^2\right)^2$ **2**

(b) Find the coordinates of the point P(x,y) which divides the interval AB externally in the ratio 1:3 with A(1,4) and B(5,2) **2**

(c) Solve the inequality $\dfrac{3}{2-x}\geq 1$ **2**

(d) Differentiate $e^{\cos^{-1}x}$ **2**

(e) Simplify $\sqrt{(\sec\theta+1)(\sec\theta-1)}$ **2**

(f) Find the Cartesian equation of the curve whose parametric equations are $x=2\cos\theta,\ y=\sqrt{3}\sin\theta,\ 0\leq\theta\leq 2\pi$ **2**

(g) Use the substitution $\sqrt{x}=u$ to evaluate $\displaystyle\int_1^9 \frac{dx}{x+\sqrt{x}}$. **3**

End of Question 11

Question 12(15 marks) Use a SEPARATE writing booklet

(a) The cubic polynomial $2x^3 + ax^2 - 7x + b$ has factors $(x-2)$ and $(x+3)$. **3**

Find the values of a and b.

(b) (i) Express $3\cos x + 2\sin x$ in the form $r\cos(x-\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $3\cos x + 2\sin x = \sqrt{13}$ for $0 \le x \le 2\pi$. Give your answer, or answers, correct to two decimal places. **2**

(c) $P(4p,\ p^2)$ and $Q(4q, q^2)$ are two points on the parabola $x^2 = 16y$.

(i) Find the coordinates of M the midpoint of PQ. **1**

(ii) If the chord PQ subtends a right angle at the origin O, show that $pq = -16$. **2**

(iii) Show that as P and Q move on the parabola the locus of M is another parabola Find its equation. **3**

(d) (i) State the domain and range of the function given by $y = \cos^{-1} 2x$. **1**

(ii) Sketch the graph of the function given by $y = \cos^{-1} 2x$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

a) From a pack of 10 cards numbered from 1 to 10, four cards are drawn at random without replacement.

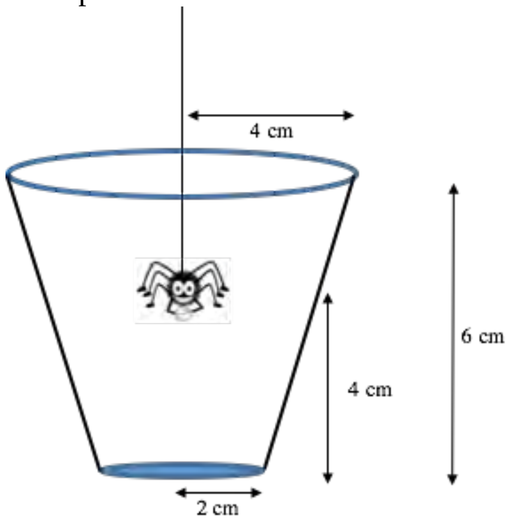
Calculate the exact probabilities that:

i) the largest number drawn is 6. **1**

ii) the product of the four numbers is even. **2**

iii) the four numbers drawn are consecutive numbers **2**

b) Water is poured at a constant rate of $20\ cm^3$ per second into a cup which is shaped like a truncated cone as shown in the figure below. The upper and lower radii of the cup are $4\ cm$ and $2cm$ respectively. The height of the cup is $6\ cm$. A spider is asleep at the end of a web hanging at $4\ cm$ vertically above the base of the cup.



i) Show that the volume of the water inside the cup, V, is related to the height of the water level, h ,through the equation **3**

$$V = \frac{\pi}{27}(h+6)^3 - 8\pi$$

ii) Find the minimum speed at which the spider must climb to avoid soaking, assuming it climbs vertically upwards the moment the water touches its feet. **2**

Question 13 continues on page 9

Question 13 (continued)

c) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds, it has: displacement x metres from a fixed point O in the line; velocity v ms^{-1} given by $v = 12\sin\left(2t + \frac{\pi}{3}\right)$ and acceleration $\ddot{x}$.

Initially the particle is 5 metres to the right of O.

(i) Show that $\ddot{x} = -4(x - 2)$ **3**

(ii) Find the period and extremities of the motion **2**

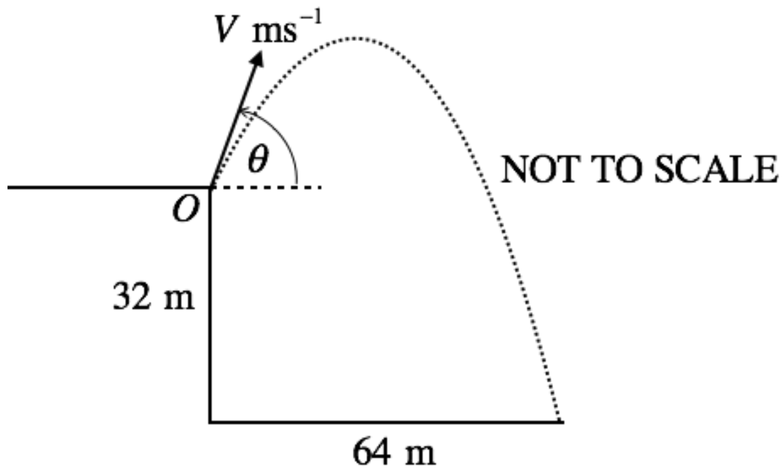
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Use mathematical induction to show that for all positive integers $n \geq 1$ **3**

$$\frac{3}{1\times2\times2}+\frac{4}{2\times3\times2^2}+.......+\frac{n+2}{n(n+1)2^n}=1-\frac{1}{(n+1)2^n}.$$

(b)



A particle is projected with velocity V ms^{-1} at an angle θ above the horizontal from a point O on the edge of a vertical cliff 32 metres above a horizontal beach.

The particle moves in a vertical plane under gravity, and 4 seconds later it hits the beach at a point 64 metres from the foot of the cliff. The acceleration due to gravity is 10 ms^{-2}.

i) Use integration to show that after t seconds the horizontal displacement x metres and the vertical displacement y metres of the particle from O are given by **2**

$$x=(V\cos\theta)t \text{ and } y=(V\sin\theta)t-5t^2 \text{ respectively}$$

ii) Write down the two equations in V and θ then solve these equations to find the exact value of V and the value of θ in degrees correct to the nearest minute. **3**

iii) Find the speed of impact with the beach correct to the nearest whole number and the angle of impact with the beach correct to the nearest minute. **3**

Question 14 continues on page 11

(c)

(i) If $\theta = \tan^{-1} A + \tan^{-1} B$ show that $\tan\theta = \dfrac{A+B}{1-AB}$ **1**

(ii) Hence solve the equation $\tan^{-1} 3x + \tan^{-1} 2x = \dfrac{\pi}{4}$. **3**

END of PAPER ☺

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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PAGE

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Extension 1 Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ (correct) C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

EX11 HSC TRIAL 2014 solutions

1A, 2C 3D 4B 5C 6C 7D 8C 9A 10B

Q11 a) $(a^2+ab)^2-(ab+b^2)^2 = (a^2+ab+ab+b^2)(a^2+ab-ab-b^2)$

$= (a^2+2ab+b^2)(a^2-b^2)$

$= (a+b)^2(a+b)(a-b)$

$= (a+b)^3(a-b)$

2 marks correctly factorises
1 mark Use difference of 2 squares, or any progress towards solution

b) A(1,4) and B(5,2)

−1 : 3

$x_P = \frac{-1(5)+3(1)}{-1+3}$, $y_P = \frac{-1(2)+3(4)}{-1+3}$

$= \frac{-5+3}{2}$ $\qquad = \frac{-2+12}{2}$

$x_P = -1$ $\qquad y_P = 5$

$\therefore P(-1,5)$

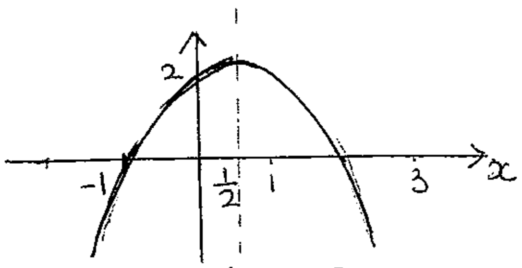
2 marks correctly evaluates P(x, y)
1 mark correctly substitute giving consideration to (−) sign.

c) $\frac{3}{2-x} \geqslant 1$ $\qquad x \neq 2$

$3(2-x) \geqslant (2-x)^2$

$(2-x)[3-(2-x)] \geqslant 0$

$(2-x)(1+x) \geqslant 0$



Solution $-1 \leqslant x < 2$

2 marks correctly evaluates
1 mark multiplies by $(2-x)^2$ or obtains one inequality for x, or any progress towards solution
2 marks other valid, correct solution

d) $\frac{d}{dx} e^{\cos^{-1}x} = e^{\cos^{-1}x} \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right)$

$u = \cos^{-1}x$

$\frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$= -\frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}$

2 marks correct answer with working out
1 mark correct differentiation of $\cos^{-1}x$,

d) $\frac{d}{dx} e^{\cos^{-1} x} = e^{\cos^{-1} x} \cdot -\frac{1}{\sqrt{1-x^2}}$

$u = \cos^{-1} x$ $\qquad = -\frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}$

$\frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}}$

e) $\sqrt{(\sec\theta+1)(\sec\theta-1)} = \sqrt{\sec^2\theta - 1}$

$= \sqrt{\tan^2\theta}$

$= \tan\theta$

2 marks correct answer with correct working out

1 mark $\sec^2\theta - 1 = \tan^2\theta$

f) $x = 2\cos\theta$, $y = \sqrt{3}\sin\theta$

$x^2 = 4\cos^2\theta$, $y^2 = 3\sin^2\theta$

$\cos^2\theta = \frac{x^2}{4}$, $\sin^2\theta = \frac{y^2}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{y^2}{3} + \frac{x^2}{4} = 1$

$3x^2 + 4y^2 = 12$

2 marks correct working and correct answer

1 mark Finding x^2 and y^2.

g) $\int_1^9 \frac{dx}{x+\sqrt{x}}$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2u}$

$x = 1 \quad u = \sqrt{1} = 1$

$x = 9 \quad u = \sqrt{9} = 3$

$dx = 2u\,du$

$\int_1^9 \frac{dx}{x+\sqrt{x}} = \int_1^3 \frac{2u\,du}{u^2+u}$

$= \int_1^3 \frac{2\,du}{u+1}$

$= 2\ln(u+1)\Big]_1^3$

$= 2(\ln 4 - \ln 2) \Rightarrow = 2\ln\frac{4}{2}$

$= 2\ln 2$

$= \ln 2^2$

$= \ln 4$

3 marks correct working out and answer

2 mark Correctly writes the integral in terms of 'u' and boundaries

1 mark any reasonable progress towards solution

12. a) Let $P(x) = 2x^3 + ax^2 - 7x + b$

$(x-2)$ is a factor of $P(x)$, $\therefore P(2) = 0$

$$2(2)^3 + a(2)^2 - 7(2) + b = 0$$
$$4a + b = -2 \quad (1)$$

$(x+3)$ is a factor of $P(x)$, $\therefore P(-3) = 0$

$$2(-3)^3 + a(-3)^2 - 7(-3) + b = 0$$
$$9a + b = 33 \quad (2)$$

Solve (1) and (2) simultaneously:

$$4a + b = -2$$
$$-\ 9a + b = 33$$
$$-5a = -35$$
$$a = 7$$

substitute $a = 7$ into (1)

$$4 \times 7 + b = -2$$
$$b = -30$$

$\therefore a = 7,\ b = -30$

3 marks correct solution

2 marks correctly evaluates $P(2)$ & $P(-3)$ and attemp to solve simultaneously

1 mark correctly evaluate $P(2)$ and $P(-3)$.

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b) i) Method I

$$3\cos x + 2\sin x = r\cos(x - \alpha)$$
$$3\cos x + 2\sin x = r\cos\alpha\cos x + r\sin\alpha\sin x$$

where $r\sin\alpha = 2 \quad (1)$

$r\cos\alpha = 3 \quad (2)$

$(1) \div (2)$ $\tan\alpha = \frac{2}{3}$, $\alpha = \tan^{-1}\frac{2}{3}$

$(1)^2 + (2)^2$ $r^2\sin^2\alpha + r^2\cos^2\alpha = 4 + 9$

$$r^2(\sin^2\alpha + \cos^2\alpha) = 13$$
$$r = \sqrt{13} \quad r > 0$$

$\therefore 3\cos x + 2\sin x = \sqrt{13}\cos\left(x - \tan^{-1}\frac{2}{3}\right)$

2 marks Correct answer

1 mark Correctly evaluates r or α, or equivalent progress.

12 b ii) $\sqrt{13}\cos(x-\tan^{-1}\frac{2}{3})=\sqrt{13}$	2 marks	correct solution
$\cos(x-\tan^{-1}\frac{2}{3})=1$	1 mark	Any progress towards solution
$x-\tan^{-1}\frac{2}{3}=0, 2\pi$		
$x=0+\tan^{-1}\frac{2}{3}$, ~~$2\pi+\tan^{-1}\frac{2}{3}$~~		
$=0.5880$, $2\pi+0.588$ (not solution)		
$=0.59$		

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c) $P(4p, p^2)$, $Q(4q, q^2)$	1 mark.	Correctly evaluates x and y.
i) $x_M=\frac{4p+4q}{2}$, $y_M=\frac{p^2+q^2}{2}$		
$=2(p+q)$		
$M\left(2(p+q), \frac{p^2+q^2}{2}\right)$		

ii) PQ subtends a right angle at the origin:	2 marks	correct answer
$\therefore PO \perp QO$ $\quad m_{PQ}\cdot m_{QO}=-1$		
$m_{PO}=\frac{p^2-0}{4p-0}=\frac{p}{4}$, $m_{QO}=\frac{q^2-0}{4q-0}=\frac{q}{4}$	1 mark	obtains m_{PO} or m_{QO}
$\therefore \frac{p}{4}\times\frac{q}{4}=-1$		
$pq=-16$		
iii) $x_M=2(p+q)$, $y_M=\frac{p^2+q^2}{2}$	3 marks	correct answer and working out
$x^2=4(p+q)^2$, $\quad p^2+q^2=2y$	2 marks	for substitution into x^2
$x^2=4(p^2+q^2+2pq)$		
$x^2=4(2y+2(-16))$	1 mark	for finding x_M, y_M
$=8(y-16)$		
$\therefore$ Locus of is a parabola, with vertex $(0, 16)$ and focus $(0, 18)$. (Focal length $4a=8$, $a=2$)		

12d) $y = \cos^{-1} 2x$

i)

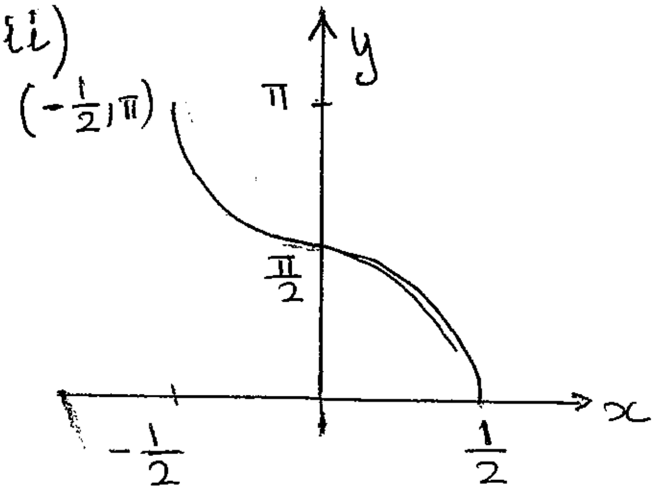
Domain $-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

Range $0 \leq y \leq \pi$

1 mark for correct domain and range

ii)



1 mark correct shape with intercept labeled

Question 13

(a) No. of ways of drawing 4 cards = $^{10}C_4$

(i) No of ways of drawing 3 cards with < 6

$= {}^5C_3$

$\therefore P(\text{largest no drawn is } 6) = \frac{{}^5C_3}{{}^{10}C_4} = \frac{1}{21}$

(ii) No of ways of drawing 4 out of 5 cards $= {}^5C_4$

$\therefore P(\text{product even}) = 1 - P(\text{all odd})$

$= 1 - \frac{{}^5C_4}{{}^{10}C_4}$

$= 1 - \frac{5}{210}$

$= \frac{41}{42}$

(iii) Possible combinations of consecutive numbers are:

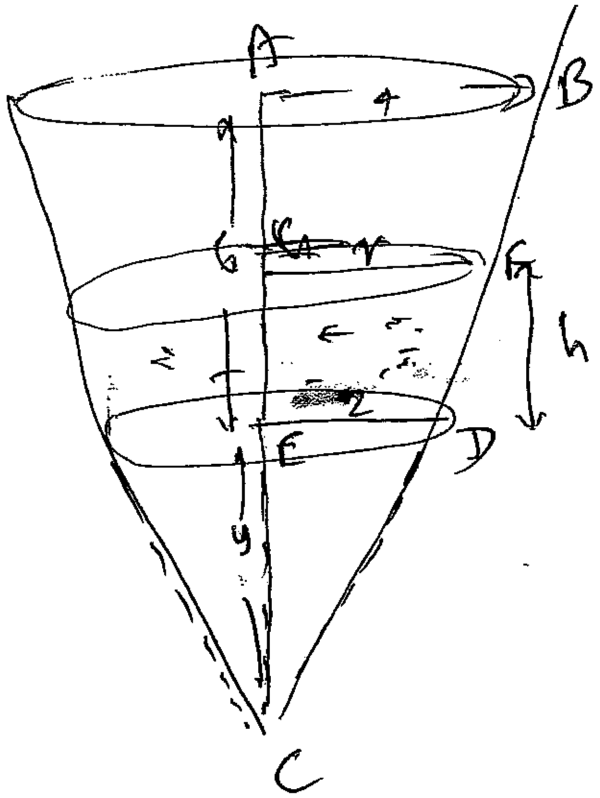
(1,2,3,4), (2,3,4,5), (3,4,5,6), (4,5,6,7), (5,6,7,8),

(6,7,8,9) and (7,8,9,10)

$\therefore P(\text{numbers consecutive}) = \frac{7}{{}^{10}C_4} = \frac{1}{30}$

Question 13

(b)



$\Delta DEC \;|||\; \Delta ABC \Rightarrow \dfrac{4}{6+y} = \dfrac{2}{y} \Rightarrow y = 6$

$\Delta GFC \;|||\; \Delta DEC \Rightarrow \dfrac{r}{h+y} = \dfrac{2}{y} \Rightarrow r = \dfrac{h+6}{3}$

Vol. of water of depth h, V

$$V = \frac{\pi}{3}\left[r^2(h+y) - 4y\right]$$

$$= \frac{\pi}{3}\left[\frac{(h+6)^2}{9}(h+6) - 4\times 6\right]$$

$$= \frac{\pi}{27}\left[(h+6)^3 - 24\right]$$

$$= \frac{\pi}{27}(h+6)^3 - 8\pi$$

Q.13 (b) (ii)

$$V = \frac{\pi}{27}(h+6)^3 - 8\pi$$

$$\frac{dV}{dh} = \frac{\pi}{9}(h+6)^2, \quad \frac{dV}{dt} = 20 \text{ cm}^3/\text{s}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

$$= \frac{9}{\pi(h+6)^2} \cdot 20$$

When water touches the spiders legs, $h = 4$

$$\Rightarrow \frac{dh}{dt} = \frac{9 \times 20}{\pi(4+6)^2}$$

$$= \frac{9}{5\pi} \text{ cm s}^{-1}$$

$\therefore$ The minimum speed that the spider must climb to avoid soaking is $\frac{9}{5\pi}$ cm s^{-1}

Question 14

(a) For $n=1$

$LHS = \frac{3}{1\times 2\times 2} = \frac{3}{4}$

$RHS = 1 - \frac{1}{2\times 2} = \frac{3}{4}$

$\therefore$ LHS = RHS for $n=1$

$\therefore$ true for $n=1$

1 for testing $n=1$ and showing LHS = RHS

Assume true for $n=k$

ie $\frac{3}{1\times 2\times 2} + \cdots + \frac{k+2}{k(k+1)2^k} = 1 - \frac{1}{(k+1)2^k}$ —— ①

1 for showing Assumption of True for $n=k$

Required to show

$$\frac{3}{1\times 2\times 2} + \cdots + \frac{k+2}{k(k+1)2^k} + \frac{k+3}{(k+1)(k+2)2^{k+1}} = 1 - \frac{1}{(k+2)2^{k+1}}$$

Now

$$LHS = \frac{3}{1\times 2\times 2} + \cdots + \frac{k+2}{k(k+1)(2^k)} + \frac{k+3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+1)2^k} + \frac{k+3}{(k+1)(k+2)2^{k+1}} \quad \text{—— from ①}$$

1 for working towards correct answer.

$$= 1 + \frac{-(k+2)2 + k+3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 + \frac{-2k - 4 + k + 3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 + \frac{-k-1}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+2)2^{k+1}}$$

= RHS $\therefore$ Proven by M. Induction.

(Note: question was wrong so students who did 1st 2 parts correct got 2 Question out of 2)

Question

(b) (i)

Horizontally

$\ddot{x} = 0$

$\dot{x} = C$

$= V\cos\theta$ (when $t=0$)

$x = V\cos\theta\, t + K$

when $t=0$, $x = 0$

$\therefore\ x = V\cos\theta\, t$

Vertically

$\ddot{y} = -10$

$\dot{y} = -10t + C$

when $t = 0$

$C = V\sin\theta$

$\therefore \dot{y} = -10t + V\sin\theta$

$y = -5t^2 + V\sin\theta\, t + K$

when $t=0$, $y = 0$

$\therefore K = 0$

$\therefore y = -5t^2 + V\sin\theta\, t$

1 for x value and 1 for y value. Must show working from basic principles

(ii)

when $t = 4$, $x = 64$

and $y = -32$

$\therefore 64 = 4V\cos\theta$ — ① and

$-32 = -5\times 4^2 + 4V\sin\theta$

$-32 = -80 + 4V\sin\theta$

$48 = 4V\sin\theta$ — ②

1 sub x, y and t to get equations

From ① $V\cos\theta = 16$ — ③

From ② $V\sin\theta = 12$ — ④

③^2 + ④^2 gives

$V^2(\cos^2\theta + \sin^2\theta) = 16^2 + 12^2$

$V^2 = 400$

$V = 20$ sub into ③ gives $\cos\theta = \frac{4}{5}$

[illegible]

1 for solving simultaneously

1 working towards correct answer

$36^\circ 52$

Question 14

(b)(iii)

when $t = 4$

$\dot{x} = V\cos\theta$ and $\dot{y} = -10t + V\sin\theta$

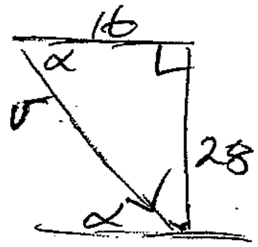
ie $V\cos\theta = 16$ (from ii) $\quad = -40 + V\sin\theta$

$\dot{x} = 16$ $\quad = -40 + 12$ (from ii)

$= -28$

Thus $-40 + V\sin\theta = -28$

1 for working out values of $\dot{x}$ and $\dot{y}$



$v^2 = 16^2 + 28^2 \implies V = 32.2$

$\tan\alpha = \frac{28}{16}$

$\alpha = \tan^{-1}\left(\frac{28}{16}\right) = 60^\circ 15'$

∴ Speed of impact is ~~60°15'~~ $32\,ms^{-1}$ at an angle of $60^\circ 15'$

1 for working out value of α

1 for working out value of θ

(c)(i) let $x = \tan^{-1}A$ and $y = \tan^{-1}B$

ie $\tan x = A$ and $\tan y = B$

and $\theta = x + y$.

1 for working towards correct answer

Now $\tan\theta = \tan(x+y)$

$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{A + B}{1 - AB}$

Question 14

(C) (ii) $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$ — ①

Now ~~from~~ using part (i): $A = 3x$

$B = 2x$

and $\theta = \frac{\pi}{4}$

$\therefore \frac{3x + 2x}{1 - (3x)(2x)} = \tan\frac{\pi}{4} = 1$

$\frac{5x}{1 - 6x^2} = 1$

$5x = 1 - 6x^2$

$6x^2 + 5x - 1 = 0$

$(6x - 1)(x + 1) = 0$

$x = \frac{1}{6}, -1$ but $x \neq -1$ (does not work in ①)

$\therefore x = \frac{1}{6}$

1 for correct quadratic

1 for eliminating $x = -1$

1 for working towards $x = \frac{1}{6}$